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Abstract. Operations Research has been a vital tool for tackling many logistical problems faced by the airline industry. Airplane logistics are focused in four main areas: schedule preparation, fleet assignment, aircraft routing, and disruption recovery. This paper focuses on the problem of aircraft routing, which involves generating and selecting a particular route for each aircraft of a sub-fleet that is already assigned to a set of feasible sequences of flight legs. Similar studies typically focus on long-term route planning. However, stochastic events such as severe weather changes, equipment failures, variable maintenance times, or even new regulations play havoc on these long-term plans. In addition, these long-term plans ignore detailed maintenance requirements by considering only one or two of the primary maintenance checks that must be performed on a regular, long-term basis. As a result, these plans are often ignored by personnel in airline operations who are forced on a daily basis to develop quick, ad hoc methods to address these maintenance requirements and other irregular events. To address this problem, we develop an operational aircraft maintenance routing problem formulation that includes maintenance resource availability constraints. We propose a direct search algorithm for solving this integer programming model.

Key words: Integer programming, Routing problem, Aircraft maintenance

1 Introduction

Operations Research has been a vital tool for tackling the many logistical problems faced by the airline industry. Airplane logistics are focused in four main areas: schedule preparation, fleet assignment, aircraft routing, and disruption recovery. In schedule preparation, the airlines prepare a list of flight legs along with departure and arrival times. This is the main product sold by airlines. This product does not specify which aircraft types are to fly which flight legs. This question is answered by the fleet assignment problem. After the fleet assignment problem has been solved, airlines face the problem of aircraft routing. Aircraft (tail number) routing assigns aircraft (tail numbers) to pre-generated routes.
Currently, these first three problems are solved well in advance, based on forecasts of customer demand and competitor supply for the various flight legs. This results in long-term plans that typically prescribe that the airplanes follow a cyclic path that revisits a maintenance station on a regular basis. However, the airline industry operates in a dynamic environment, where many unplanned events force airlines to modify their existing long-term plans. Such unexpected events or disruptions include severe weather changes, unplanned equipment failures, and emergency maintenance requests from the FAA and aircraft manufacturers. Disruption recovery is the process of reacting to these operational disruptions. The decision-making in disruption recovery is the process of rerouting in order to recover aircraft, crew, and passengers. In this dynamic environment, the airlines try to stay competitive by responding to these changes quickly while keeping their daily operations running as smoothly as possible. Doing so is quite a challenge, particularly because the complex logistical routing plans are developed long-term, and unexpected events that upset these plans occur daily. This leaves airline personnel with the daily challenge of adjusting long-term plans to fit immediate operational constraints posed by these unexpected events.

Although there is much literature that deals with aircraft routing on a long-term planning basis, there is little work on the more pressing operational routing problem faced daily by the airlines. We address this weakness by considering the problem of operational aircraft maintenance routing. This means that instead of developing optimal long-term routing techniques, we intend to develop methods to enable airlines to perform short-term aircraft routing in the face of irregular operations.

This paper is organized in the following fashion. Section 2 presents a brief literature review of aircraft fleeting, routing and disruption recovery. Section 3 discusses the daily operational aircraft maintenance routing problem and its characteristics. A mathematical formulation of the problem is given in Section 4. Section 5 explains the proposed direct search method. Finally, in Section 6 we provide some concluding remarks.

2 Literature Review

A rich body of literature addresses airline problems (fleet assignment, aircraft routing, yield management, etc.) and the use of operations research tools for these problems. The studies related to the problem at hand may be classified into three groups, fleeting-related studies, routing-related studies, and disruption recovery studies. The following paragraphs briefly describe some of these major studies.

The earlier research primarily focused on the classical fleet assignment problems. Abara (1989) explained the classical fleet assignment problem,
which included constraints on flight coverage, equipment continuity, aircraft count, and some operational constraints such as gate availability, noise restriction, etc. Subramanian et al. (1994) introduced a large-scale mixed integer linear programming (MILP) formulation, COLDSTART, for the classical fleet assignment model including some maintenance and crew pairing constraints along with the concept of one-stop flights and passenger spill costs.

Hane et al. (1995) formulated and solved a daily fleet assignment problem as an MILP, Clarke et al. (1996) extended the study in Hane et al. (1995) using some maintenance and crew considerations. The maintenance requirements were classified into two groups, short-term maintenance (A-check) and long-term maintenance (B-check), based upon the time requirements of the maintenance. Using the same reduction methods in Hane et al. (1995), the authors solved this problem within two to five hours. The solutions obtained in both studies in Clarke et al. (1996) and Hane et al. (1995) were aggregate maintenance feasible fleet assignments, not necessarily a maintenance feasible aircraft tail number routing. Moreover, both studies have no evidence of any daily operational resource constraints.

While some researchers focused on the fleet assignment problem as listed above, other researchers studied aircraft routing problems, which assume that the fleet assignment problem is already solved. Daskin and Panayotopoulos (1989) formulated a route selection problem to maximize profits in a single hub-and-spoke network as an MILP. Feo and Bard (1989) presented a model used by American Airlines (AA) both to locate maintenance stations and to develop flight schedules that better met cyclical demand for A-checks. Kabani and Patty (1993) presented an A-check feasible route selection model, which was formulated as a set-partitioning problem.

Clarke et al. (1997) presented a mathematical formulation for the aircraft rotation problem in which a specific route for each aircraft was determined under the maintenance feasibility constraints. This formulation bears many similarities to the asymmetric traveling-salesman problem. Gopalan and Talluri (1998) developed a polynomial-time algorithm for long-term aircraft maintenance routing that employed a route-modification scheme of swaps and interchanges among aircraft to form a feasible solution.

More recently, two studies have combined fleet assignment and aircraft routing. These two problems were traditionally solved separately due to their large size. Ignoring maintenance constraints allowed Desaulniers et al. (1997) to reduce the size of the problems and develop two models for a combined fleeting and routing problem called the Daily Aircraft Routing and Scheduling Problem (DARSP). Barnhart et al. (1998) also developed a model to solve both fleeting and routing problems simultaneously. The authors attempted to solve these two problems together to overcome the traditional drawback of a
classical fleet assignment problem: the lack of a well-spaced maintenance opportunity for each aircraft of the sub-fleet.

In the disruption recovery area, there are some related studies in the literature. Some of these earlier studies include Teodorovic and Guberinic (1984) and (1990), Jarrah et al. (1993), Cao and Kanafani (2000). These studies focused on certain aspects of disruption recovery such as minimizing the customer delay, minimizing the number of aircraft needed to recover from a disruption, minimizing the number of cancelled flight legs, minimizing the schedule perturbations, and maximizing the profit with consideration for cancellation and delay cost penalties. These studies are primarily considered to be “aircraft recovery” studies. There are other studies, Lettovsky et al (2000) and Stojkovic (1998), for example, that focus on “crew recovery” instead. The current trend in disruption recovery is on building integrated models in which aircraft and crew recovery are considered together. Lettovsky (1997) presented a model for integrated recovery in which crew, aircraft routing, and passenger recovery was considered all together. Recently, Rosenberger (2002) provides a comprehensive review of the related literature for integrated (hybrid) modeling. Despite the similarity of these many studies to our problem, none have addressed our problem. Most of these studies are for (long-term) planning purposes rather than (short-term or daily) operational purposes except for the disruption recovery studies. The few studies that are operational in nature fail to consider the legal remaining flying hours of the aircraft and the resource constraints at the maintenance stations. It is with this in mind we address the daily operational aircraft maintenance routing problem in the next section.

3 Problem Definition and Characteristics

The components of an aircraft may be grouped into two main classes, resident and non-resident components (Bandla, 1994). Resident components consist of the aircraft and its vital systems and are grouped for maintenance purpose according to their functionalities. These groups are maintained on a regular basis via letter checks. For example, the A-check (performed after 65 hours of flying) involves visual inspection of the aircraft and its vital systems (Clarke et al., 1997). On the other hand, non-resident components (call-outs) such as an engine master chip detector need to be maintained individually. Thus, each non-resident component has its own maximum legal remaining flying hour limit.

For safety considerations, the Federal Aviation Administration (FAA) mandates these legal remaining flying hour limits. Violations result in severe financial penalties. Most airlines employ automated routing systems to keep track of the accumulated flying hours for both letter checks and for non-resident components of aircraft. When the accumulated flying hours of an
aircraft reach a predefined level, the aircraft is labeled as a “high-time” aircraft and is listed in the daily list of high-time aircraft. An aircraft may appear more than once in this list due to multiple maintenance requirements. The aircraft on the daily high-time list need to be routed to feasible maintenance stations where there exist enough man-hours and maintenance slots to perform the required maintenance.

Fig. 1. Time limit definitions for an aircraft.

In practice, the task of routing high-time aircraft for maintenance involves two major steps. In the first step, multiple maintenance requirements of a high-time aircraft are batched into a package. The legal remaining flying hours of this package is defined as the minimum of the legal remaining flying hours of the maintenance requirements in the package. In the second step, all aircraft of a particular sub-fleet are routed in such a way that all flight legs are covered and high-time aircraft end up at a feasible maintenance station. Airlines perform these two main steps using some heuristic methods to optimize various performance measures.

These heuristic methods do not always find a feasible solution to the problem at hand, however. The infeasibility occurs because a high-time aircraft is forced to stay overnight at a station where there does not exist either enough man-hours or necessary equipment to perform the maintenance. To continue the daily operations and avoid severe financial penalties, airlines are forced to move their maintenance personnel and equipment from station to station, at substantial additional cost.

The daily aircraft maintenance routing is a problem in which the objective is to minimize the total daily maintenance costs without violating legal remaining flying hours of each aircraft, subject to the resource constraints (available man-hours and maintenance slots) of the maintenance stations. The optimal solution of this problem will not only minimize total daily maintenance costs, but also reduce the movement of maintenance personnel and equipment from one
station to another. Because it is difficult to define and calculate a daily maintenance cost for an aircraft, we utilize a surrogate objective to accomplish our primary goal. As a surrogate objective, we minimize the total cushion time (total unutilized legal remaining flying hours) of a sub-fleet. In other words, we intend to maximize the utilization of the total green time (total legal remaining flying hours) for a sub-fleet. (See Fig. 1.)

A feasible solution is one in which all high-time aircraft are routed to a feasible maintenance station for the given day. The maintenance for all high-time aircraft is assumed to happen at the end of the day during the overnight stay. If an aircraft requires a longer maintenance than can be achieved overnight, then enough maintenance time can be ensured by either selecting its ending flight leg to finish early in the day or its starting flight leg for the following day to begin late in that day.

The time horizon of the problem is assumed to be one day. (The horizon may in fact be shorter than a day since a disruption might occur in the middle of the day, necessitating a recovery for the remainder of the flight leg network with the given start locations for the aircraft.) A time horizon longer than one day, say one week, becomes more of a planning horizon that is too optimistic for the frequent disruptions that plague the airline industry. The solution to a problem with a longer time horizon may indeed better utilize the remaining flying hours of the aircraft by delaying the maintenance of some high-time aircraft beyond the current day but still within the legal limits. These opportunities are relatively rare, however, since the airlines impose fairly tight maintenance time limits, even more strict than what the FAA requires. With a longer time horizon, one could also argue for including “non-high-time” aircraft, i.e., those aircraft with remaining legal flying hours of two days or more. However, maintaining these aircraft earlier than necessary is counterproductive to the goal of maximizing the utilization of aircraft green time and consumes scarce maintenance slot and man-hour resources needed for the high-time aircraft.

4 Problem Formulation

We start by explaining the network structure that will form the basis for the mathematical formulation of our problem.

4.1 Network Structure

Airline flight schedules are traditionally depicted as flight-leg networks, where nodes represent cities and arcs between nodes represent flight legs connecting these cities. A difficulty with this representation involves keeping track of the departure and arrival times of each arc in the network. To remove this
difficulty, we use a connection network structure in which nodes represent flight legs and arcs represent feasible connections among the flight legs. In this so-called connection network, the presence of an arc \((i, j)\) between node (flight leg) \(i\) and node (flight leg) \(j\) means that the departure city of node \(j\) is the same as the arrival city of node \(i\), and the arrival time of flight leg \(i\) plus the turn time (the time needed to prepare the aircraft for the next flight) is less than or equal to the departure time of flight leg \(j\). In other words, if the same aircraft can successively fly flight legs \(i\) and \(j\) then arc \((i, j)\) must exist. It is trivial to show that the connection network is always acyclic (Sarac, 2000).

The connection network includes dummy source and sink nodes that are connected to appropriate starting and ending flight legs, respectively. The arcs from the dummy source to the other nodes in the network have a flight duration of zero, while the arcs to the dummy sink node will have a duration of the incoming flight leg (node). Furthermore, the underlying flight schedule of the connection network must be balanced, which means the number of outbound flights of a particular city/airport must equal the number of inbound flights of the same city (Clarke et al., 1997).

4.2 Mathematical Formulation

We use a set-partitioning based formulation in which decision variables represent feasible routes for aircraft. We choose this approach for several reasons: it can easily incorporate route-based constraints, it focuses on the feasible assignment of routes to aircraft, and such formulations have proven to be very promising for general vehicle routing problems (Barnhart et al., 1998; Desrosiers et al., 1984; Desrochers et al., 1989; Desrochers et al., 1992; Dumas et al., 1991; Krishnamurthy, 1990; Ryan and Foster, 1981; Savelsbergh and Sol, 1998; Vance et al., 1997). Table 1 summarizes the notation for our formulation.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>Index for flight legs.</td>
</tr>
<tr>
<td>(j)</td>
<td>Index for routes.</td>
</tr>
<tr>
<td>(m)</td>
<td>Index for maintenance types.</td>
</tr>
<tr>
<td>(k)</td>
<td>Index for aircraft.</td>
</tr>
<tr>
<td>(s)</td>
<td>Index for overnight stations.</td>
</tr>
<tr>
<td>(o)</td>
<td>Dummy source node used in sub-problem formulation.</td>
</tr>
<tr>
<td>(t)</td>
<td>Dummy sink node used in sub-problem formulation.</td>
</tr>
<tr>
<td>(A)</td>
<td>Set of connection arcs in sub-problem formulation.</td>
</tr>
</tbody>
</table>
The cost coefficients, $c_j^k$, associated with decision variables (routes) represent the cushion time (Figure 1) for the aircraft. If aircraft $k$ is not a high-time aircraft, then the corresponding cost coefficients of all its routes take zero value. On the other hand, if aircraft $k$ is a high-time aircraft, then $c_j^k$ is equal to the legal remaining flying hours of the aircraft $k$ minus the duration of route $j$. The decision variable, $c_j^k$, represents a feasible route $j$ (string of flight legs) for the aircraft $k$ of a particular sub-fleet. Moreover, there are four sets of constraints: aircraft coverage (1), flight leg coverage (2), man-hour availability (3), and slot availability (4). The general mathematical formulation becomes:
(0) \[ \min \sum_{k \in K} \sum_{j \in R_k} c_{kj} y_{kj} \] 

(subject to)

(1) \[ \sum_{j \in R_k} y_{kj} = 1 \quad \forall k \in K \] \hspace{1cm} (\pi_k)

(2) \[ \sum_{k \in K} \sum_{j \in R_k} y_{kj} = 1 \quad \forall i \in N \] \hspace{1cm} (\sigma_i) \hspace{1cm} (P)

(3) \[ \sum_{k \in K} \sum_{j \in R_k} a_{mk} d_{kj} y_{kj} \leq L_{ms} \quad \forall m \in M \text{ and } s \in S_m \] \hspace{1cm} (\beta_{ms})

(4) \[ \sum_{k \in K} \sum_{j \in R_k} b_{mk} d_{kj} y_{kj} \leq Z_{ms} \quad \forall m \in M \text{ and } s \in S_m \] \hspace{1cm} (\alpha_{ms})

(5) \[ y_{kj} \in \{0,1\} \quad \forall k \in K \text{ and } j \in R_k \]

The above formulation aims to minimize the number of unused legal flying hours while ensuring aircraft count (1) and flight leg coverage (2) by maintaining feasibility for available maintenance man-hours (3) and the number of maintenance slots (4). The necessity of using two separate constraints for the maintenance hangar capacity is due to the possibility of having some man-hours but no slots at some maintenance station or vice versa. This situation forces airlines to make undesirable costly maintenance personnel moves from station to station.

The problem (P) has an exponential number of feasible routes and cannot be solved directly in an efficient fashion. In fact, this problem is NP-hard since it contains the NP-complete partition problem as a special case (Sarac, 2000).

5 The Direct Search Algorithm

After solving the relaxed problem, the procedure for searching a suboptimal but integer-feasible solution from an optimal continuous solution can be described as follows.

Let \[ x = [x] + f, \quad 0 \leq f \leq 1 \]

be the (continuous) solution of the relaxed problem, \([x]\) is the integer component of non-integer variable \(x\) and \(f\) is the fractional component.

Step 1. Get row \(i^*\) the smallest integer infeasibility, such that \[ \delta_{i^*} = \min\{f_j, 1-f_j\} \]

Step 2. Calculate \[ v_{i^*} = c_{i^*} B^{-1} \]

this is a pricing operation
Step 3. Calculate $\sigma_{ij} = v_j^T a_j$

With $j$ corresponds to $\min_j \left| \frac{d_j}{\sigma_{ij}} \right|$

I. For nonbasic $j$ at lower bound

If $\sigma_{ij} < 0$ and $\delta_* = f_i$ calculate $\Delta = \frac{1 - \delta_*}{-\sigma_{ij}}$

If $\sigma_{ij} > 0$ and $\delta_* = 1 - f_i$ calculate $\Delta = \frac{1 - \delta_*}{\sigma_{ij}}$

If $\sigma_{ij} < 0$ and $\delta_* = 1 - f_i$ calculate $\Delta = \frac{\delta_*}{\sigma_{ij}}$

If $\sigma_{ij} > 0$ and $\delta_* = f_i$ calculate $\Delta = \frac{\delta_*}{\sigma_{ij}}$

II. For nonbasic $j$ at upper bound

If $\sigma_{ij} < 0$ and $\delta_* = 1 - f_i$ calculate $\Delta = \frac{1 - \delta_*}{-\sigma_{ij}}$

If $\sigma_{ij} > 0$ and $\delta_* = f_i$ calculate $\Delta = \frac{1 - \delta_*}{\sigma_{ij}}$

If $\sigma_{ij} > 0$ and $\delta_* = 1 - f_i$ calculate $\Delta = \frac{\delta_*}{\sigma_{ij}}$

If $\sigma_{ij} < 0$ and $\delta_* = f_i$ calculate $\Delta = \frac{\delta_*}{-\sigma_{ij}}$

Otherwise go to next non-integer nonbasic or superbasic $j$ (if available). Eventually the column $j^*$ is to be increased from LB or decreased from UB. If none go to next $i^*$.

Step 4. Calculate $\alpha_{j^*} = B^{-1} a_{j^*}$

i.e. solve $B \alpha_{j^*} = \alpha_{j^*}$ for $\alpha_{j^*}$.

Step 5. Ratio test; there would be three possibilities for the basic variables in order to stay feasible due to the releasing of nonbasic $j^*$ from its bounds.

If $j^*$ lower bound

Let

$$A = \min_{i\neq j^*} \left\{ \frac{x_{j^*} - l_i}{\alpha_{j^*}} \right\}$$
\[
B = \min_{i' \in A | x_{i'} < 0} \left\{ \frac{u_{i'} - x_{B_{i'}}}{-\alpha_{ij}} \right\}
\]

\[
C = \Delta
\]

the maximum movement of \( j^* \) depends on:

\[
\Theta^B = \min(A, B, C)
\]

If \( j^* \) upper bound

Let

\[
A' = \min_{i' \in A | \alpha_{ij} < 0} \left\{ \frac{x_{B_{i'}} - l_{i'}}{\alpha_{ij}} \right\}
\]

\[
B' = \min_{i' \in A | \alpha_{ij} > 0} \left\{ \frac{u_{i'} - x_{B_{i'}}}{-\alpha_{ij}} \right\}
\]

\[
C' = \Delta
\]

the maximum movement of \( j^* \) depends on:

\[
\Theta^B = \min(A', B', C')
\]

step 6. Exchanging basis for the three possibilities

1. If \( A \) or \( A' \)
   - \( x_{B_{i'}} \) becomes nonbasic at lower bound \( l_{i'} \)
   - \( x_{j^*} \) becomes basic (replaces \( x_{B_{i'}} \))
   - \( x_{i} \) stays basic (non-integer)

2. If \( B \) or \( B' \)
   - \( x_{B_{i'}} \) becomes nonbasic at upper bound \( u_{i'} \)
   - \( x_{j^*} \) becomes basic (replaces \( x_{B_{i'}} \))
   - \( x_{i} \) stays basic (non-integer)

3. If \( C \) or \( C' \)
   - \( x_{j^*} \) becomes basic (replaces \( x_{i} \))
   - \( x_{i} \) becomes superbasic at integer-valued

repeat from step 1.

6 Concluding Remarks

We have described a direct search approach to solving an operational aircraft routing problem that faces airlines on a daily basis. This problem is unique in the literature in that it takes an operational, rather than long-term planning, view of aircraft maintenance routing. Further, we incorporate resource availability constraints into the model, which cannot be ignored when realistically solving an operational problem.
References